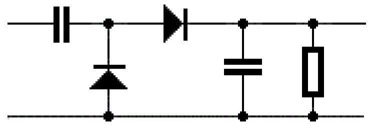
Diode-pump



Introduction into mathematics needed for calculus of the diodepump

1. Introduction

The diode-pump is pretended an insignificant-simple electronic circuit, which only consists of 5 components, but the complete description, what it really is in all details, is a long story. It got a big meaning in the 1960’s in the radiation-physics, where radioactive radiation, coming as particles into a Geiger-counter, causing there electronic metering-pulses, which could be used, among others, to measure thickness of material, because intensity of through-coming radiation was, due to the properties of penetrating radiation, an exponential function of the thickness. Logarithmizing with the voltage-current-characteristic of diodes implicated a high temperature-dependence and buildups where made for temperature-stabilising, but problems were great and the diode-pump had no temperature-dependence. The diode-pump works similar as a water- or air-pump, pumping a medium into a tank, which has a hole at the bottom, where the medium is running out again. The faster you pump, the higher the filling-level in the tank. The both diodes of the diode-pump work similar as the valves of an air-pump, they are only pervious in one direction. The practical buildup and measurement of the diode-pump’s properties delivers sufficient values for the needed frequency-response-function in dependence of the chosen components in order to get a proper dimensioning. But how can it be understood mathe-matically and got calculated specifications with optimizing linearity-errors and can be given exact information about the measurement-error ?

Extra made for the beginners, all theory starts, from far off from the matter, with the quadratic equation and how their roots can be found, *because in the quadratic equation is the origin, from where imaginary numbers come into the equations of electronic circuits serving there as symbols for the characteristic, physical properties of electronic components. Because the quadratic equation is the root of the differential-equation of the electric oscillation-circuit and the quadratic equation has occasionally, depending on the special values of their coefficients, imaginary numbers as roots, which occur then in the exponents of the exponential-functions of the differential-equation, what guides us to the periodic oscillations. And all this is extra done only to shed light on the notion of impedance, which is extra used to show the principle of conversion from a real-circuit to an equivalent-circuit, in order to proof, that the equivalent-circuit is justifiable to be used for all kinds of periodic and aperiodic signals. Because the equivalent-circuit is necessary to make all formulas simpler. And that is finally the completed pre-condition for starting to set up the calculus-equations for the diode-pump, which is shown in the next contribution.*

The only function, which can serve as a solution of the differential equation is the exponential function, because e^x after differentiation remains e^x and falls out of the equation to make it possible, that the sum of the equation can get zero, while the differentiation-coefficients $(a.e^{kx})'' = a.k^2.e^{kx}$, $(b.e^{kx})' = b.k.e^{kx}$ …. form a quadratic equation. But because, the solution of a differential equation is a quadratic equation, the roots are occasionally imaginary numbers. If you really understand, what happens exactly with the charge, if 2 in series connected condensers are loaded up combined, you have got the precondition for forming an equivalent-circuit with capacities.

The following mathematics tells you the story from the very beginning of the fundamental facts of electronics until you get at the end out of it the frequency-response-function of the diode-pump. To make all easier, parts of the complex functions are replaced and expressed by symbols (that means: left as they are) and equations with the symbols are reduced to its simplest form. The end-formula looks comparatively simple, but does not say much in its hidden complexity, but the calculation of numerical values can be simply made by writing all formulas into an excel-table and making a diagram of it. Today it is no longer necessary to calculate out complex functions by hand. After this you can philosophize, why the frequency-response is approximately logarithmic in a certain range. I think, that it comes from the circumstance, that the higher the input-frequency, the less charge is loaded in per each pulse.

2. Roots of the quadratic equation

   here as preparation for the coming ponderings the derivation is shown how to come to the roots of a quadratic equation, because they can contain imaginary numbers and they are later in the exponents of the differential equation causing periodic oscillations.

   If we have a quadratic equation of this form:

$x^2 + px + q = 0$ we don't know the roots, but

for the following equation we know the roots as x_1 and x_2 ,

because for $x = x_1$ and $x = x_2$ the equation $= 0$

$$(x - x_1)\cdot(x - x_2) = 0 = x^2 - (x_1 + x_2)\cdot x + x_1 x_2$$

but the parameters are different, therefor we set:

$$-(x_1 + x_2) = p \quad \text{and} \quad x_1 x_2 = q$$

in order to express x_1 now explicitly we go to eliminate x_2

$$x_2 = -p - x_1$$

$$x_1 x_2 = q = -x_1(p + x_1) \qquad \Rightarrow \qquad x_1^2 + p\cdot x_1 = -q$$

new we add an expression to the equation to eliminate $p\cdot x_1$:

$$\underbrace{x_1^2 + p\cdot x_1 + \left(\frac{p}{2}\right)^2} = \left(\frac{p}{2}\right)^2 - q$$

$$\left(x_1 + \frac{p}{2}\right)^2 = \left(\frac{p}{2}\right)^2 - q$$

after square root of the equation :

$$x_1 + \left(\frac{p}{2}\right) = \pm\sqrt{\left(\frac{p}{2}\right)^2 - q} \qquad \Rightarrow \qquad x_1 = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

because x_1 and x_2 can be swappedin this prodecure, we get :

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q} \quad \Rightarrow \quad x_1 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q} \qquad x_2 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}$$

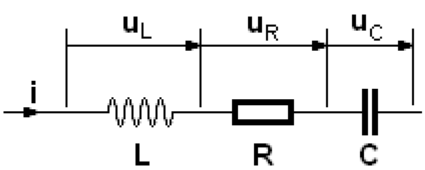
If we have the equation $ax^2 + bx + c = 0$ instead of $x^2 + px + q = 0$
we need only todevide equation $ax^2 + bx + c = 0$ by a and
replace the factors by p and q

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \text{and set} \quad p = \frac{b}{a} \quad \text{and } q = \frac{c}{a}$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q} = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. The differential equation of the electronic oscillation-circuit as also shown, though, in another context on the website:

http://erich-foltyn.eu\ Technique\DiffEqu1.html



but here shall be shown the seamless transition from a time-function to an endless oscillation by variation of the coefficients in order to show the role of imaginary numbers in the electronic occurances.

$$u_L + u_R + u_C = 0$$

$$L\frac{di}{dt} + iR + \frac{1}{C}\int i\,dt = 0 \qquad 1)$$

$$L \cdot i'' + R \cdot i' + \frac{1}{C} i = 0 \qquad \text{after differentiation of 1)}$$

$$a \cdot i'' + b \cdot i' + c \cdot i = 0$$

$i = i_0 \cdot e^{\lambda t}$ we insert by way of trial for i(t) the only possible function $e^{\lambda t}$ with the unknown constant λ

$$a \cdot i_0 \cdot \lambda^2 \cdot e^{\lambda t} + b \cdot i_0 \cdot \lambda \cdot e^{\lambda t} + c \cdot i_0 \cdot e^{\lambda t} = 0$$

$$\left(a \cdot \lambda^2 + b \cdot \lambda + c\right) \cdot i_0 \cdot e^{\lambda t} = 0 \qquad 2)$$

$$a\lambda^2 + b \cdot \lambda + c = 0 \quad \text{for} \quad L\lambda^2 + R\lambda + \frac{1}{C} = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{that is}$$

$$\lambda_{1,2} = \frac{-R \pm \sqrt{R^2 - 4\frac{L}{C}}}{2L} = \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{4\frac{LR^2}{CR^2}}{4L^2}} = \frac{-R}{2L}\left(1 \mp \sqrt{1 - 4 \cdot \frac{1}{RC} \cdot \frac{L}{R}}\right)$$

that numbers 2 and 4 come from, that C and L share the same resistance R for their time constants $\frac{L}{R}$ and RC, that is the time, when the voltage has decayed to the $\frac{1}{e}$ – fold of the inital - value.

Now it is clear, that in the equation 2) the quadratic equation makes the entire equation always zero for $\lambda_{1,2}$ independently from the special value of $e^{\lambda t}$ if $\lambda = \lambda_1$ or λ_2 to deliver by the differentiation of $e^{\lambda t}$ the appropriate coefficients for the quadratic equation to have these roots $\lambda_{1,2}$. And as closer described on my website:

http://erich-foltyn.eu/Technique/DiffEqu2.html

the input-function of the differential equation looks anyhow like:

$$i = i_0 \cdot \left(e^{\lambda_1 t} + e^{\lambda_2 t} \right) \quad \text{etc.}$$

in case of $4 \frac{1}{RC} \cdot \frac{L}{R} > 1$ we get $\sqrt{-1}$ in the exponent and as we know:

$$j = \sqrt{-1} \qquad e^{\pm j\varphi} = \cos\varphi \pm j \cdot \sin\varphi \qquad \mathrm{e}^{\alpha \pm \mathrm{j}\beta} = e^{\alpha} \cdot (\cos\beta \pm j \cdot \sin\beta)$$

the evidence for this you can find in the powerseries of e^x and $\cos x, \sin x$

$$\mathrm{e}^{\mathrm{x}} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \quad \text{and we set } x = jy$$

$$\mathrm{e}^{\mathrm{jy}} = 1 + \frac{jy}{1!} + \frac{(jy)^2}{2!} + \frac{(jy)^3}{3!} + \ldots$$ do we seperate the real and imaginary parts:

$$\mathrm{e}^{\mathrm{jy}} = \left(1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \frac{y^6}{6!} + \ldots \right) + j \left(\frac{y}{1!} - \frac{y^3}{3!} + \frac{y^5}{5!} - \frac{y^7}{7!} + \ldots \right)$$

and these are the series for cosine and sine

now we see, that if any constant β:

if $\beta \cdot \mathrm{t} = 2 \cdot \pi$ then $\beta = 2 \cdot \pi \cdot \frac{1}{t} = 2 \cdot \pi \cdot f = \omega$

therefor we can say in case of $4 \frac{1}{RC} \cdot \frac{L}{R} > 1$

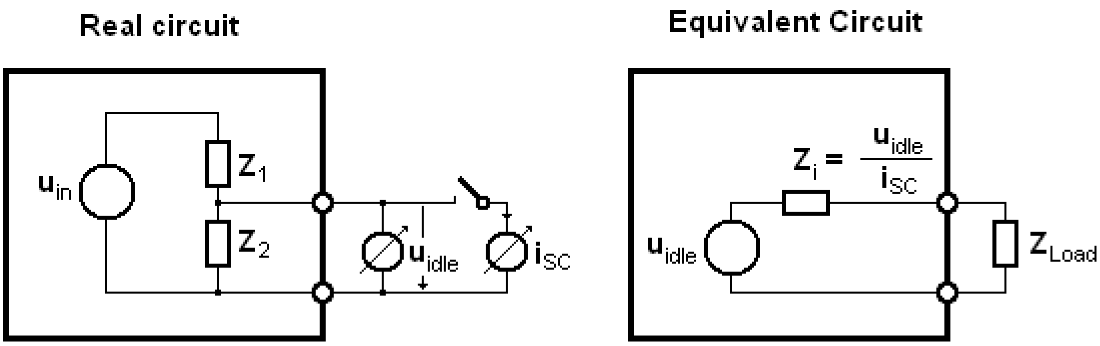
$$\frac{-R}{2L} \sqrt{1 - 4 \cdot \frac{1}{RC} \cdot \frac{L}{R}} = j \cdot \omega$$

in case of a coil and a condenser, the quotient of voltage and current is the impedanz Z, that is an imaginary number and right now we have seen, how an imaginary number can play a role as a part of the physical magnitude of electronic components.

$$\frac{u}{i} = Z \qquad Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C}$$

we need this in order to say something about the equivalent circuits

4. Equivalent Circuit



u_{idle} ... idle-voltage

i_{SC} ... short-circuit-current

Z_1, Z_2 ... impedances

Z_i ... inner impedanz of the equivalent circuit's current source

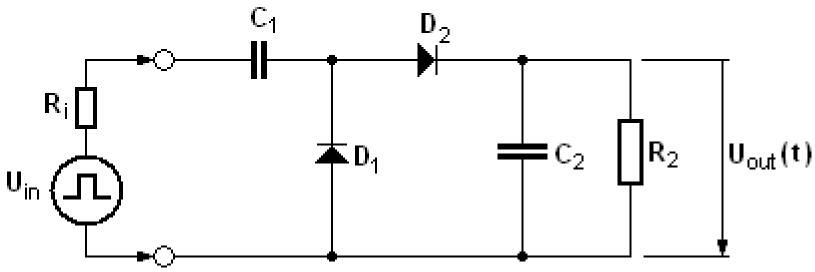
u_{in} ... f (t), g (ω)

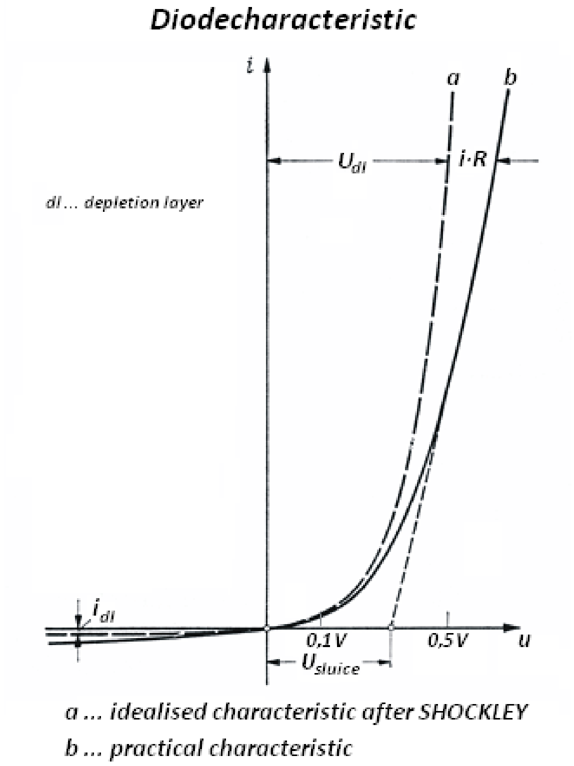
in case of u_{in} is alternating voltage and measuring of u_{idle} and i_{SC} by instruments, the phasing between them must be considered

the impedance $Z = \frac{u(t)}{i(t)} = R + jX$, X is reactance

$$u_{idle} = u_{in}\frac{Z_2}{Z_1 + Z_2} \qquad i_{SC} = \frac{u_{in}}{Z_1} \qquad Z_i = \frac{u_{idle}}{i_{SC}} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

an that is the parallel - connection of Z_1 and Z_2





$$i = i_{dl} \cdot \left(e^{-\frac{u_{dl}}{c \cdot u_T}} - 1 \right)$$

i_{dl} depletion – layer – current (strongly temperatur-dependent)

u_{dl} voltage at the depletion layer, positive in forward-direction

$u_T = \dfrac{kT}{e}$ temperature-voltage (25mV by 290 degrees K)

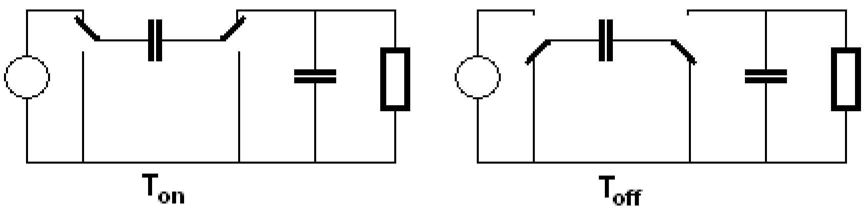
T temperature in degrees K

e elementary charge

c 1 to 2

Here you see the characteristic of the diodes. But for the simplified mathematical treatment one thinks the diodes as ideal switches, which have in switched-on-state the forward-resistance null, in the switched-off-state infinite and the time null for switching over. It is namely impossible to let the exponential current-voltage-characteristics of the diodes flow-in into the equations as a non-linear resistance.

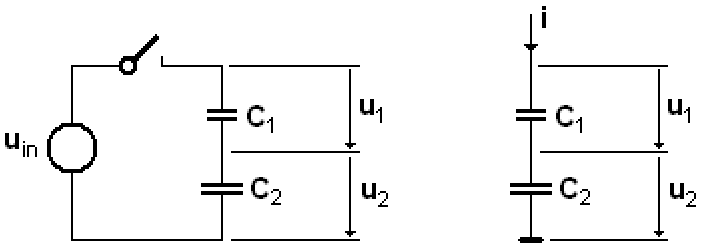
The principle of the



mathematical ansatz is the following:

f is the pulse-frequency and the time T = 1/f is those time, after which the preceeding state repeats: u(t) = u(t+T), that is the steady state, after which one has long enough fed-in the pulse-frequency. The charge, which flows in the time T from the pulse-current-source into the condensers C_1 and C_2, is the same charge, which flows in the same time out over the resistor R_2 again. Both charges Q_{in} und Q_{out} are a function of the direct-voltage U_2 at C_2, so that it adjusts itself the preconditioned balance-state by this.

Flowing-in charge:



Flows current from the pulse-source through the in serie connected condensers C_1 and C_2 over a certain time, so is flown the same amount of charge into both (Q = i . t) . Thereafter must be the formula Q = C . U likewise valid for the total system as Q = C . U for every single condenser:

$$Q_1 = C_1 \cdot U_1 \qquad Q_2 = C_2 \cdot U_2 \qquad Q_{total} = Q_1 = Q_2$$

$$Q_{total} = \frac{C_1 \cdot C_2}{C_1 + C_2} \cdot (U_1 + U_2) = C_1 \cdot U_1 = C_2 \cdot U_2$$

$$\frac{U_1}{U_1 + U_2} = \frac{C_2}{C_1 + C_2} \qquad \frac{U_2}{U_1 + U_2} = \frac{C_1}{C_1 + C_2}$$

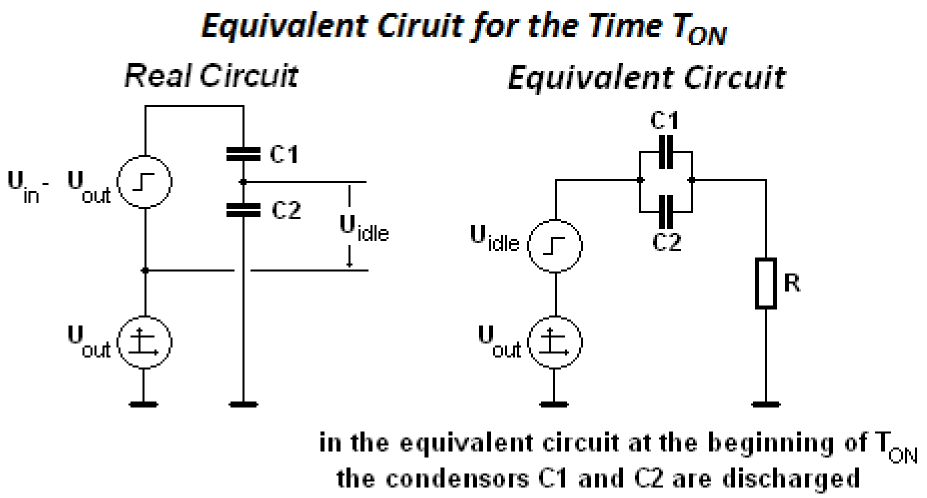
In the factual circuit there is the condensor C_2 charged up to the voltage $U_{2,t=0}$ and we connect the discharged condensor C_1 in serie and charge then both up to the voltage U_{in} . For understanding, what happens, we assume, that we, instead of it, feed-in a constant current i so long into the both condensors, until the sum of their voltages $U_1 + U_2$ has the value U_{in} .

$$U_{in} = U_1 + U_2 = \left(U_{2,t=0} + \frac{\mathrm{i} \cdot \mathrm{t}}{\mathrm{C}_2} \right) + \frac{\mathrm{i} \cdot \mathrm{t}}{\mathrm{C}_1}$$

$$\mathrm{i} \cdot \mathrm{t} = \mathrm{Q}$$

$$U_{in} - U_{2,t=0} = Q \cdot \left(\frac{1}{\mathrm{C}_2} + \frac{1}{\mathrm{C}_1} \right)$$

One sees from it, that seen from the flowing-in charge it is the same, as if both condensers would be discharged and one puts on the voltage $U_{in} - U_{2,t=0}$ at them. By the flowing-out current into the resistor R_2 it looks certainly something different, because this is determined by $U_{2,t=0}$. Because an equivalent circuit for easier mathematic treatment is a simplification of the factual relations, but one must first examine, if their functions are in ful accordance with the factual circuit. By an equivalent circuit we assume, that it has at its output the same idle-voltage u_{idle} and the same short-circuit-current i_{SC} as the factual circuit. The idle-voltage will be generated in the equivalent-circuit by an equivalent-voltage-source, which has the inner resistance null and u_{idle} / i_{SC} yields the inner resistance of the equivalent-circuit.



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